

Hong Kong Mathematics Olympiad (1988 – 89)

Heat Event (Group)

香港数学竞赛 (1988 – 89)

初赛项目 (团体)

1. Given a and b are distinct real numbers satisfying $a^2 = 5a + 10$ and $b^2 = 5b + 10$. Find the value of $\frac{1}{a^2} + \frac{1}{b^2}$.

a 、 b 为两相异实数，且 $a^2 = 5a + 10$ 及 $b^2 = 5b + 10$ ，求 $\frac{1}{a^2} + \frac{1}{b^2}$ 的值。

2. An interior angle of an n -sided convex polygon is x° while the sum of other interior angles is 800° . Find the value of n .

一凸 n 边形的一个内角是 x° ，其他内角的和是 800° ，求 n 的值。

3. It is known that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n .

Find the value of $21^2 + 22^2 + \cdots + 30^2$.

已知对所有正整数 n ， $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ ，求 $21^2 + 22^2 + \cdots + 30^2$ 的值。

4. One of the positive integral solutions of the equation $19x + 88y = 1988$ is given by $(100, 1)$. Find another positive integral solution.

方程 $19x + 88y = 1988$ 的其中一组正整数解是 $(100, 1)$ ，求另一组正整数解。

5. The line joining $A(2, 3)$ and $B(17, 23)$ meets the line $2x - y = 7$ at P . Find the value of $\frac{AP}{PB}$.

$A(2, 3)$ 与 $B(17, 23)$ 的连线交 $2x - y = 7$ 于 P ，求 $\frac{AP}{PB}$ 的值。

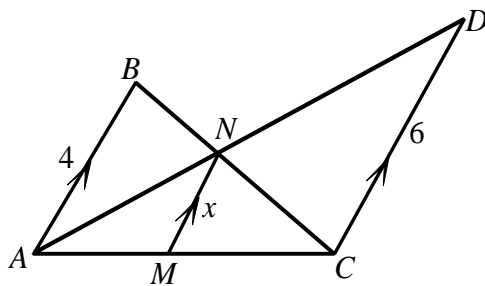
6. Find the remainder when 7^{2047} is divided by 100.

求 7^{2047} 被 100 除所得的余数。

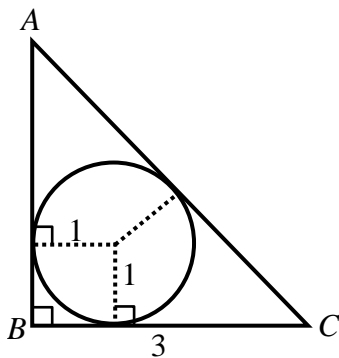
7. If $\log_2[\log_3(\log_7 x)] = \log_3[\log_7(\log_2 y)] = \log_7[\log_2(\log_3 z)] = 0$, find the value of $x + y + z$.

若 $\log_2[\log_3(\log_7 x)] = \log_3[\log_7(\log_2 y)] = \log_7[\log_2(\log_3 z)] = 0$ ，求 $x + y + z$ 的值。

8. In figure 1, $AB \parallel MN \parallel CD$. If $AB = 4$, $CD = 6$ and $MN = x$, find the value of x .
 在图 1 中, $AB \parallel MN \parallel CD$ 。若 $AB = 4$ 、 $CD = 6$ 及 $MN = x$, 求 x 的值。



9. In Figure 2, $\angle B = 90^\circ$, $BC = 3$ and the radius of the inscribed circle of $\triangle ABC$ is 1. Find the length of AC .
 在图 2 中, $\angle B = 90^\circ$ 、 $BC = 3$, 且 $\triangle ABC$ 的内切圆半径长 1 单位, 求 AC 的长。



10. In the attached division (see Figure 3), the dividend in (a) is divisible by the divisor in line (b). Find the dividend in line (a). (Each asterisk * is an integer from 0 to 9.)
 在所附除法算式中 (见图 3), (a) 列的被除数可被 (b) 列的除数整除。求 (a) 列的被除数。(每一星号 * 为 0 至 9 的整数。)

$$\begin{array}{r}
 \text{(b) } \dots * * * \overline{) \begin{array}{r} * * * * * * * \dots \text{(a)} \\ * * * * \\ \hline * * * \\ * * * \\ \hline * * * * \\ * * * * \\ \hline \end{array}}
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(Figure 3)(图 3)